ECE 174 Homework # 3 — Due Thursday, 11/9/2017

There are ten (10) questions on this homework assignment. Remember, the Solutions Manual is provided with the text. Errata for the textbook are available at the website:

http://MatrixAnalysis.com/Errata.html

Reading From Chapter 5

Read sections §5.1; §5.3; §5.4 (ignore Example 5.4.3); §5.9; §5.11 (through page 406 inclusive); and §5.13.

The reading for this course is very important. For example, the proof of the "reverse triangle inequality" shown in Example 5.1.1 is exactly the kind of proof I might put on an exam. Further, insights provided in the reading are very illuminating.

Thus Example 5.4.6 shows that a function $f(t)$ can be viewed as an element of a vector space which can be expanded (using Equation 5.4.3) in terms of basis vectors (basis functions) which are sines and cosines. ***This shows that the theory of Fourier series is a kind of generalization of Linear Algebra*** and helps to ***demystify*** the mathematics underlying Fourier series.[1]

Reminder of First Computer Assignment Due Date - Tuesday 11/7/2017

Remember, the first computer assignment (with written report) is due Tuesday, November 7, 2017, which is the lecture just prior to the due date for Homework 3. Therefore stay on top of the computer project and do NOT wait until the last minute to do it.

Reminder of Midterm Date - Tuesday 11/14/2017

★ The Midterm is scheduled for Tuesday, November 14, 2017. (7th week of the quarter.)

Homework

1. Prof. Vasconcelos asked his students the following question in ECE175: Consider the matrix
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}.$$
   Answer these questions *immediately, merely from visual inspection* of the matrix:

   i) What is the column space of the matrix? ii) What is its row space? iii) What is its nullspace? iv) What is its rank? v) What is the dimension of its column space? vi) What is a basis for its nullspace?

[1] In ECE175A, a homework problem on this is often given.

2. 5.1.8, 5.1.9, 5.1.10.

3. 5.3.2.

4. 5.4.14.

5. 5.9.5.

6. 5.11.5.

7. 5.13.3.

8. Using the abstract definition of an inner product on a complex hilbert space, prove the following properties.[2] (The complex conjugate of a scalar α is denoted by $\overline{\alpha}$.)

   a) $\langle x_1, \alpha x_2 \rangle = \langle \overline{\alpha} x_1, x_2 \rangle$.
   b) $\langle \alpha_1 x_1 + \alpha_2 x_2, x \rangle = \overline{\alpha}_1 \langle x_1, x \rangle + \overline{\alpha}_2 \langle x_2, x \rangle$.

9. Let $A : \mathcal{X} \to \mathcal{Y}$, $B : \mathcal{X} \to \mathcal{Y}$, and $C : \mathcal{Y} \to \mathcal{Z}$ be linear mappings as shown between complex finite–dimensional hilbert spaces $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$. Let α and β be complex numbers with complex conjugates $\overline{\alpha}$ and $\overline{\beta}$. As discussed in lecture, the adjoint operator, A^*, is defined by

$$\langle A^* x_1, x_2 \rangle = \langle x_1, A x_2 \rangle \ .$$

   Using the abstract definition of an inner product on a complex hilbert space prove the following properties.[3]

   a) $(\alpha A)^* = \overline{\alpha} A^*$.
   b) $(A + B)^* = A^* + B^*$.
   c) $(\alpha A + \beta B)^* = \overline{\alpha} A^* + \overline{\beta} B^*$.
   d) $A^{**} = A$.
   e) $(CA)^* = A^* C^*$.
   f) $A^* \alpha y = \alpha A^* y$.
   g) $A^*(y_1 + y_2) = A^* y_1 + A^* y_2$.
   h) $A^*(\alpha_1 y_1 + \alpha_2 y_2) = \alpha_1 A^* y_1 + \alpha_2 A^* y_2$.

   Note that property (h) shows that the adjoint operator A^*, like A itself, is also a linear operator.

10. Let $A : \mathcal{X} \to \mathcal{Y}$ be an $m \times n$ linear mapping between two hilbert spaces as shown. Let

$$\langle x_1, x_2 \rangle = x_1^H \Omega x_2 \quad \text{and} \quad \langle y_1, y_2 \rangle = y_1^H W y_2 \, .$$

[2] If on an exam you can only do this for the special case where $\langle x_1, x_2 \rangle = x_1^H \Omega x_2$, $\Omega^H = \Omega > 0$, then you will lose many points. (Note that this special case corresponds to the hilbert space being finite dimensional with its elements represented canonically, i.e., as column vectors.)

[3] See the previous footnote.

(a) Derive the adjoint operator A^* in terms of A, Ω, and W.

(b) For $\mathrm{r}(A) = n$, derive the pseudoinverse of A and show that it is independent of the weighting matrix Ω.

(c) For $\mathrm{r}(A) = m$, derive the pseudoinverse of A and show that it is independent of the weighting matrix W.